Theory and Practice of Science and Technology
2026, VOL. 7, NO. 1, 100-103
DOI: 10.47297/taposatWSP2633-456921.20260701

A Review of Stochastic Dominance-Based Route Choice Models and Algorithms

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ABSTRACT
Travel time stochasticity in transportation networks is a key factor driving route choice behavior. Conventional models fail to account for heterogeneous risk preferences and travel time reliability, showing significant practical limitations. Stochastic Dominance (SD) theory offers a unified, utility-free framework for uncertain route choice via partial order characterization of random travel times, and has gained extensive attention in traffic optimization. This paper systematically reviews SD-based route choice models and algorithms, sorts out model construction under different network environments, summarizes core algorithm design and optimization strategies, and clarifies their applicability. Finally, it summarizes research limitations and prospects future directions, providing reference for relevant theoretical research and engineering applications.

KEYWORDS
Stochastic dominance; Route choice optimization; Stochastic network optimization; Travel time reliability; Solution algorithm

1 Introduction

In urban transportation networks, link travel times exhibit stochastic characteristics due to congestion, weather fluctuations, traffic incidents, and other factors. When deciding on routes, travelers not only focus on expected travel times but also display varying sensitivities to travel time reliability and delay risks. Traditional shortest path models, such as Dijkstra's algorithm, prioritize minimizing expected travel time without considering travel time variability. Meanwhile, route choice models relying on single reliability indicators—like on-time arrival probability or percentile travel time—struggle to capture the inherent differences in individual risk preferences.

Originating from economics and decision science, stochastic dominance theory establishes partial order relations among alternatives through comparisons of their cumulative distribution functions (CDFs). Its core advantage lies in its ability to characterize decision preferences under different risk attitudes without requiring prior specification of utility function forms. Wu and Nie [1] were the first to introduce first-order (FSD), second-order (SSD), and third-order (TSD) stochastic dominance theory into traffic route choice research, proposing the concept of SD-admissible paths alongside foundational modeling methods. Subsequent research has expanded along three key dimensions: network characteristics evolving from static to time-varying stochastic networks, research objects extending from individual route choice to network traffic assignment, and solution methods advancing from classical dynamic programming to fusion optimization with intelligent algorithms. These developments have collectively formed a coherent research thread encompassing "model formulation-algorithm design-scenario application."

Centering on models and algorithms, this review systematically organizes the formulation logic and core equations of FSD/SSD/TSD route choice models, summarizes principal solution algorithms for static and time-varying stochastic networks, and analyzes their applicability and limitations. Our goal is to provide a clear analytical framework for future research in this area.

2 Theoretical Foundations of Stochastic Dominance and Its Applicability to Traffic Route Choice

This chapter presents scenario-adapted definitions of FSD, SSD, and TSD dominance rules tailored to traffic route choice contexts, clarifies the core concept of SD-admissible paths and their hereditary properties, and elaborates on how SD-based models unify and extend traditional reliability-based route choice models.

2.1 Core Dominance Rules Defined for Traffic Scenarios[1]

Let π_k represent the random travel time of path k, with a cumulative distribution function $F_k(t) = P(\pi_k \le t)$, and let T denote the upper bound of travel time. To align with the traffic decision-making principle that "shorter travel time is preferred," Wu and Nie [1] defined the three orders of SD rules as follows:

(1) First-Order Stochastic Dominance (FSD): For all $t \in [0,T]$, $F_k(t) \ge F_l(t)$ holds, with strict inequality in a non-empty interval $\Lambda \subseteq [0,T]$ (of non-zero Lebesgue measure); in this case, path k is said to first-order stochastically dominate path l, denoted as $k \succ_1 l$. FSD corresponds to risk-neutral or greedy travelers, who prioritize paths with higher on-time arrival probabilities at any given time threshold.

(2) Second-Order Stochastic Dominance (SSD): For all $t \in [0,T]$, $\int_t^T F_k(w)dw \ge \int_t^T F_l(w)dw$ holds, with strict inequality in a

non-empty interval $\Lambda \subseteq [0,T]$; here, path k second-order stochastically dominates path l, denoted as $k \succ_2 l$ .SSD is suited for risk-averse travelers, who dislike travel time variability and tend to select paths with smaller expected delays.

(3) Third-Order Stochastic Dominance (TSD):For all $t \in [0,T]$, $\int_t^T \int_w^T F_k(v)dvdw \geq \int_t^T \int_w^T F_l(v)dvdw$ holds, path k thus third-order stochastically dominates path l, denoted as $k \succ_3 l$.TSD caters to extremely risk-averse travelers—such as those on emergency trips—who are highly sensitive to extreme delays and prefer paths with lower tail risks.

2.2 Core Concept of SD-Admissible Paths

For the path set K_{rs} of an origin-destination (OD) pair (r,s), a path $k \in K_{rs}$ is deemed an SD-admissible path of a specific order (FSD/SSD/TSD) if no other path $l \in K_{rs}$ satisfies $l \succ_d k (d = 1,2,3)$. The SD-admissible path sets for each order are denoted as Γ_{FSD}, Γ_{SSD} and Γ_{TSD}, respectively, and satisfy the inclusion relationship $\Gamma_{TSD} \subseteq \Gamma_{SSD} \subseteq \Gamma_{FSD}$ [1].

SD-admissible path sets serve as decision boundaries for travelers with different risk preferences: any traveler's optimal path will necessarily belong to the SD-admissible path set corresponding to their risk attitude [1]. This property simplifies model solutions significantly, as it eliminates the need to enumerate all possible paths—instead, only filtering the SD-admissible path set suffices to identify optimal decisions.

2.3 Relationship with Traditional Reliability-Based Route Choice Models

SD-based route choice models can be seen as a unification and extension of traditional reliability models. Specifically: FSD dominance is equivalent to achieving higher on-time arrival probability across all confidence levels, aligning with the core objectives of on-time probability (OTP) and percentile travel time (PTT) models; SSD models avoid the need to prespecify the punctuality parameter λ in travel time budget (TTB) models, enabling natural characterization of risk-averse behavior [1][9]; and TSD models address the shortcoming of traditional models in capturing extreme delay aversion [1][17].

Nikolova and Stier-Moses [9] further noted that SD models provide a theoretical foundation for mean-risk models, with dominance rules functioning as key constraints:

$$\min_{k \in K_{rs}} E[\pi_k] + \theta \cdot Risk(\pi_k), \qquad s.t. \qquad k \in \Gamma_{SSD}$$

where $Risk(\pi_k)$ denotes a risk measure (such as variance) and θ is a risk weighting coefficient.

3 Stochastic Dominance-Based Route Choice Models

SD-based route choice models are categorized into static and time-varying stochastic network models, with the latter emerging as the mainstream direction of current research. The model system has been further enriched by extensions for multi-constraint and multi-objective scenarios, as well as localized models tailored to preference-based and emergency travel needs [14-17].

3.1 SD Route Choice Models in Static Stochastic Networks

Static stochastic networks mark the starting point of SD-based route choice research, with the core objective of establishing FSD/SSD/TSD-based route choice criteria and filtering SD-admissible path sets [1-2].

(1) Basic SD-Admissible Path Choice Model:Wu and Nie [1] framed the route choice problem as filtering SD-admissible path sets using non-dominance constraints, pioneering the connection between SD theory and traffic route choice.

(2) Optimal Path Model with SSD Constraints:To address the decision-making needs of risk-averse travelers, Nie et al. [2] converted SSD rules into linear programming constraints, developing an optimal path model focused on minimizing expected travel time:

$$\begin{aligned} &\min_{k \in K_{rs}} E[\pi_k] \\ &s.t. \qquad \int_t^T F_k(w)\,dw \geq \int_t^T F_l(w)\,dw, \qquad \forall t \in [0,T], \forall l \in K_{rs} \end{aligned}.$$

(4) Mean-Standard Deviation SD Model: Wu [3] integrated mean-standard deviation criteria with SSD dominance to construct a bi-objective optimization model:

$$\begin{aligned} &\min_{k \in K_{rs}} \{E[\pi_k], Var(\pi_k)\} \\ &s.t. \qquad k \in \Gamma_{SSD} \end{aligned}.$$

This model overcomes the limitation of traditional mean-variance models, which are only applicable to normally distributed data .

3.2 SD Models in Time-Varying Stochastic Networks

In actual transportation networks, link travel time distributions evolve dynamically over time, forming time-varying stochastic networks that typically satisfy the stochastic first-in-first-out (S-FIFO) property [5]. Route choice models in such contexts must account for both temporal dynamics and stochasticity, making this area a core focus of current research [6][7][8][14].

(1) Reliable SD Model under S-FIFO Property: Chen et al. [5] adapted FSD rules to accommodate the S-FIFO property

(where travel time distributions are denoted as $F(t|s) = P(\pi(s) \le t)$, dependent on departure time s), with the core constraint:

$$F_k(t|s) \ge F_l(t|s), \forall t \in [0,T], \forall s \in [s_0, s_1].$$

Here, $[s_0, s_1]$ represents the feasible departure time window. Building on this, Chen et al. [6] introduced the on-time arrival probability as a reliability indicator, constructing a TSD model with reliability constraints:

$$\begin{aligned} &\min_{k \in K_{rs}} v_k(\alpha|s) \\ &s.t. \quad \int_t^T \int_w^T F_k(v|s) dv dw \ge \int_t^T \int_w^T F_l(v|s) dv dw, \forall t \in [0,T], \forall l \in K_{rs}. \end{aligned}$$

In this formulation $v_k(\alpha|s)$ denotes the α –percentile travel time at departure time s.

(2) SD Model in Dynamic Stochastic Networks: Wu [7] extended time-varying stochastic networks to dynamic stochastic networks (where travel time distributions are updated with real-time traffic flow, denoted as $F(t|\tau) = P(\pi(\tau) \le t)$ with τ as the current time,establishing the core constraint:

$$F_k(t|\tau) \ge F_l(t|\tau), \forall t \in [0,T], \forall \tau \in [\tau_0, \tau_1].$$

(3) Constrained Reliable SD Model: Ruß et al. [14] incorporated practical hard constraints (such as distance D_k and cost C_k) into their model:

$$\begin{aligned} &\min_{k \in K_{rs}} E[\pi_k(s)] \\ &s.t. \quad \int_t^T F_k(w|s) dw \ge \int_t^T F_l(w|s) dw. \\ &\forall t \in [0,T], \forall l \in K_{rs}, D_k \le D_{\max}, C_k \le C_{\max} \end{aligned}$$

Here, D_{max} and C_{max} represent the upper bounds for distance and cost, respectively.

3.3 Extensions for Multi-Objective, Multi-Preference, and Network-Level Scenarios

Rajabi-Bahaabadi et al. [15] developed a TSD-constrained multi-objective model considering both travel time and cost, employing the Non-dominated Sorting Genetic Algorithm (NSGA-II) to generate Pareto-optimal path sets. Chen and Xu [16] integrated travelers' arterial road affinity into FSD-admissible path selection via weighted utility functions, enabling SD frameworks to account for subjective preferences. Wei and Liu [17] designed a TSD-based emergency routing model aimed at maximizing on-time arrival probability under strict deadlines.

Nikolova and Stier-Moses [9] embedded SSD constraints into user equilibrium conditions, elevating SD theory from individual route choice to the network traffic assignment level [10-11]. Schmidt et al. [12] further generalized this approach through strategy dominance, comparing complete traveler strategies—including departure time and route sequences—using SSD relations. These extensions highlight SD theory's flexibility in adapting to diverse decision contexts while upholding rigorous risk-theoretic foundations.

4 Solution Algorithms for Stochastic Dominance-Based Route Choice

The primary goal of solving SD-based route choice problems is to efficiently filter SD-admissible path sets. Direct enumeration of all paths for dominance testing is computationally infeasible, as the number of paths in a network grows exponentially with the number of nodes [1,4]. All existing algorithms are grounded in dynamic programming and have been optimized around four key directions: SD-admissible path generation, dominance test acceleration, dynamic network adaptation, and intelligent algorithm fusion [7,15].

(1) Basic SD-admissible path generation algorithms: These algorithms generate SD-admissible path sets through recursive path extension and dominance verification. The Label-Correcting Algorithm (SD-LC) [1] stands as a classic benchmark for static networks, adopting a backward extension strategy from the destination and leveraging the hereditary property of SD-admissible sub-paths to avoid full path enumeration. For uncertain networks with unknown travel time distributions, Aljubayrin et al. [4] proposed a non-dominated path filtering algorithm that combines FSD dominance with non-dominated sorting, defines interval-specific FSD dominance criteria, and employs a greedy strategy for path selection—addressing the SD-LC algorithm's reliance on precise travel time CDFs.

(2) Acceleration algorithms for dominance tests and convolution computation: The core computational bottlenecks lie in the convolution of path travel time CDFs and integral-based dominance testing for SSD/TSD. Wu [7] introduced the Discrete Fourier Transform (DFT) to convert time-domain convolution into frequency-domain multiplication, reducing computational complexity from $O(t^2)$ to $O(t \log t)$—a method now widely adopted as the standard for convolution acceleration in SD algorithms for time-varying stochastic networks. Additionally, Wu and Nie [1] simplified SSD/TSD dominance testing using the integral lemma, transforming multi-dimensional integrals into point-wise calculations of travel time thresholds. This innovation avoids complex multiple integral operations and boosts testing efficiency by over 30%, serving as the foundational criterion for subsequent SSD/TSD algorithms [2,6,17].

(3) SD algorithm optimization for time-varying/dynamic stochastic road networks: Adapting to travel time dynamics and meeting real-time engineering requirements remain key challenges . Chen et al. [5] extended the SD-LC algorithm to a time-varying label-correcting algorithm for the S-FIFO property, expanding path labels to include departure time-travel time joint CDFs and enabling dynamic generation of SD-admissible path sets for different departure periods; a reliability

threshold pruning strategy was later added to reduce computational load [6]. For dynamic stochastic networks, Wu [7] proposed an incremental update algorithm for SD-admissible path sets, classifying network links as stable or dynamic and only updating CDFs and conducting dominance tests for paths associated with dynamic links—improving real-time response speeds by over 50%. For constrained models, Ruß et al. [14] integrated a constraint pruning strategy into the SD-LC algorithm, eliminating paths that fail to meet hard constraints before conducting dominance tests and enhancing computational efficiency by 40% in constrained scenarios.

(4) Intelligent algorithm fusion for multi-objective scenarios: Traditional dynamic programming struggles to handle multi-objective SD models, which involve both multi-objective optimization and SD dominance testing. To address this, researchers have integrated metaheuristic algorithms with SD dominance rules to develop specialized intelligent solutions. The NSGA-II-SD fusion algorithm [15] stands as a core representative: it adopts SD dominance rules as the primary non-dominated sorting criterion for NSGA-II, combining the global search capabilities of metaheuristic algorithms with SD's risk characterization strengths. This approach effectively solves multi-objective SD route choice problems and provides a reference paradigm for integrating intelligent algorithms with SD theory.

5 Conclusion and Outlook

Stochastic Dominance (SD) theory establishes a unified risk modeling framework for route choice under uncertain traffic conditions, accurately capturing travelers' heterogeneous risk preferences through different order rules and addressing the limitations of conventional models [18]. To date, this field has developed a comprehensive model system spanning static and time-varying networks (from individual to network-level applications) and a high-efficiency algorithm system built around the SD-LC algorithm. Relevant findings have been successfully applied in scenarios such as intelligent navigation and emergency traffic management. However, key limitations persist: insufficient model generalization, suboptimal algorithm real-time performance, and poor adaptability to engineering implementation.

Future research can advance along four critical directions: in-depth model expansion, lightweight algorithm innovation, multi-source data-driven optimization, and engineering application. By leveraging multidisciplinary integration, intelligent technology advancements, and vehicle-infrastructure cooperation, future work is expected to overcome existing limitations, enhance the real-world applicability of models and algorithms, and provide sustained theoretical and technical support for urban traffic network optimization, personalized mobility services, and emergency traffic control.

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